

**SOME RESULTS ON $(0, 2)$ Z_N ORBIFOLD
THEORIES WITH CONTINUOUS WILSON LINES***

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ABSTRACT

We discuss recent results on orbifold compactifications with $(0,2)$ world sheet supersymmetry and continuous Wilson lines, emphasizing the role of modular symmetries.

1. Introduction

Orbifold compactifications ^{1,2,3} of the heterotic string are interesting to study, because they are both exactly solvable and give rise to a $N = 1$ supersymmetric spectra with interesting gauge groups and chiral matter. Whereas for a general compactification one needs $(2, 2)$ world sheet (WS) supersymmetry in order to compute some quantities of interest, one can consider here models with $(0, 2)$ WS supersymmetry which is sufficient in order to have $N = 1$ space time (ST) supersymmetry. If the model is constructed such that some components of the Wilson line can still be varied continuously, then the rank of the gauge group can be smaller than 16 ⁴ and its level can be bigger than 1 ⁵, thus allowing reasonable gauge groups and a realistic Higgs sector for GUT models. The physical properties can be worked out using the effective supergravity action valid below the Planck or string scale. Here one uses the fact there is a one to one correspondence between the moduli parametrizing deformations of the string theory and scalars with flat potential in the effective action. Thus the generalized kinetic term of these scalars is fixed by the Kähler potential of the moduli metric. The global structure of the moduli space is also reflected by the effective action which must be invariant under the modular group of the internal sector. This can be used ⁶ once the Kähler potential K is known in order to restrict

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the form of the superpotential W , because for example the effective potential does only depend on the combination $G = K + \log |W|^2$.

2. Results

2.1. Local structure of moduli space, Kähler potentials

The moduli of an orbifold model fall into two classes: the untwisted and the twisted moduli. The untwisted moduli are those moduli of the original model, which are compatible with the twist and therefore are still moduli of the twisted model. The twisted moduli, which we will not study here, live in the so called twisted sectors that one has to add in order to preserve WS modular invariance.

In the case of Narain compactifications the moduli space is locally isomorphic to $SO(22,6)/SO(22) \otimes SO(6)$ ⁷. $SO(22,6)$ is the group of allowed deformations of the Narain lattice (which parametrizes the moduli dependent part of the theory) and the subgroup of Euclidean rotations is modded out, because its action is physically trivial. If one now constructs an asymmetric orbifold by modding out a rotation $\Theta = \Theta_L \otimes \Theta_R \in SO(22) \otimes SO(6)$, then the space of untwisted moduli is locally given by the subgroup of $SO(22,6)$ commuting with Θ modulo the subgroup of pure rotations. As shown in⁸ the result is

$$\mathcal{M}_O(\Theta) = \bigotimes_{i=1}^n \frac{SU(p_i, q_i)}{SU(p_i) \otimes SU(q_i) \otimes U(1)} \otimes \frac{SO(r, s)}{SO(r) \otimes SO(s)} \otimes \frac{SO(u, v)}{SO(u) \otimes SO(v)} \quad (1)$$

where p_i, r, u and q_i, s, v are the multiplicities of the eigenvalues $e^{2\pi i \phi_i}, -1, 1$ of Θ_L and Θ_R respectively. If Θ_R fulfills the constraint $SU(2) \not\supset \Theta_R \in SU(3)$ imposed by $N = 1$ ST supersymmetry², then $v = 0$ and either $s = 0$ or $s = 2$ and the moduli space is a complex Kähler manifold. The Kähler potential can now be found along the lines of^{8,9}. In the limit of vanishing Wilson lines the results of^{10,11} are reproduced. Let us now look at the effect of continuous Wilson lines in a concrete example and assume that the internal torus factorizes as $\mathbf{T}^6 = \mathbf{T}^2 \otimes \mathbf{T}^4$ such that the internal twist acts as $-I_2$ on \mathbf{T}^2 .

In this case there is a T and a U modulus as usual^{12,11}, which are now supplemented by two continuous Wilson lines associated with the inequivalent directions on the torus, taking values in that part of the Cartan subalgebra of the gauge group on which the gauge twist acts as $-I$. Assuming for definiteness that this subspace has also dimension two, we have four real Wilson moduli which can be rearranged into two complex ones, B and C . According to the general result the moduli space is now

$$\mathcal{M}_{4,2} = \frac{SO(4,2)}{SO(4) \otimes SO(2)} \text{ and } K = -\log \left((T + \bar{T})(U + \bar{U}) - \frac{1}{2}(B + \bar{C})(C + \bar{B}) \right) \quad (2)$$

is the Kähler potential as shown in⁸. For vanishing Wilson lines the moduli space factorizes into two separate $SU(1,1)/U(1)$ cosets corresponding to complex and to Kähler deformations. Thus in this limit, where the $(0,2)$ WS supersymmetry is enhanced to $(2,2)$ special geometry is restored. The holomorphic mixing terms are

physically interesting because terms of this type can generate once local supersymmetry is spontaneously broken a Higgs mass term of order $m_{3/2}$ thus offering a solution to the μ problem¹³. As observed in^{14,15} such terms appear naturally through matter fields in $(2, 2)$ compactifications.

2.2. Modular symmetries

Moduli spaces of string compactifications have a very non-trivial global structure because some large deformations are actually automorphisms^{16,12,17}. This is analog to moduli spaces of Riemann surfaces. The modular groups of $SO(r, 2)$ and $SU(m, n)$ cosets are denoted by $SO(r, 2, \mathbf{Z})$ and $SU(m, n, \mathbf{Z})$, but note that they depend on the details of the underlying lattices and therefore some care is required in their definition. For compactifications on $\mathbf{T}^2/\mathbf{Z}_2$ with two complex Wilson line moduli we find that the well known subgroup $SL(2, \mathbf{Z})_T$ of the modular group acts by

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad U \rightarrow U - \frac{ic}{2} \frac{BC}{icT + d}, \quad B \rightarrow \frac{B}{icT + d}, \quad C \rightarrow \frac{C}{icT + d} \quad (3)$$

with $a, b, c, d \in \mathbf{Z}$, $ad - bc = 1$. Note that the same transformation law also applies to matter field $(2, 2)$ compactifications¹⁵. As is evident from the so called mirror symmetry $T \leftrightarrow U$, there exists another subgroup $SL(2, \mathbf{Z})_U$ with an obvious action on the moduli.

There are also modular transformations which are only non-trivial when the Wilson lines are switched on. These contain translations of the Wilson lines by lattice vectors¹⁸ which have the following action on the complex moduli⁸:

$$T \rightarrow T + pU + \frac{p}{2}\sqrt{2}(B + C), \quad U \rightarrow U, \quad B \rightarrow B + p\sqrt{2}U, \quad C \rightarrow C + p\sqrt{2}U, \quad (4)$$

with $p \in \mathbf{Z}$. All these transformations leave the Kähler potential invariant up to a Kähler transformation.

2.3. Modular Forms and Superpotentials

Finally we want to use the transformation properties of the Kähler potential to construct some candidates for non-perturbative superpotentials along the lines of¹⁹. The starting point is the mass formula for string states in a \mathbf{Z}_2 sector of an $N = 1$ orbifold, which we will choose to be the one related to an $SO(4, 2)$ coset and with Wilson lines taking values in the CSA of a $SU(2)^2$ ⁹:

$$\alpha' M^2 = \frac{|\mathcal{M}|^2}{Y} + 2(N - 1 + l^T l + m^T n) \quad (5)$$

The first term on the right hand side is moduli dependent and contains the holomorphic mass \mathcal{M} ¹⁹ and a real analytic function Y which is related to the Kähler potential by $K = -\log Y$. The second term is moduli independent and contains the number N of left moving excitations together with the vectors $l = (l_i)$, $n = (n_i)$ and $m = (m_i)$,

$i = 1, 2$ of charge, winding and momentum quantum numbers. The term $l^T l + m^T n$ is proportional to the $SO(4, 2)$ invariant scalar product. Setting the second term to zero gives an $SO(4, 2)$ invariant constraint which restricts the quantum numbers to live in a certain orbit \mathcal{O} of $SO(4, 2, \mathbf{Z})$. Comparing the mass formula to the form of the G function, one learns that $W_{\mathcal{O}}$, where $\log W_{\mathcal{O}} = \sum_{\mathcal{O}} \log \mathcal{M}$, is a candidate for a non-perturbative superpotential because it transforms formally in the right way. Since the sums in general diverge some regularization is needed. Two interesting orbits are given by $\mathcal{O}_1 : N = 1 \Rightarrow l^T l + m^T n = 0$ and $\mathcal{O}_0 : N = 0 \Rightarrow l^T l + m^T n = 1$. Here, we will only consider the following simplest $SO(2, 2, \mathbf{Z})$ -invariant suborbits, namely $\mathcal{O}_{1,0} : l = 0, m^T n = 0$ and $\mathcal{O}_{0,1}, \dots, \mathcal{O}_{0,4} : l^T l = 1, m = n = 0$, which give rise to three inequivalent functions,

$$W_1 = \eta^{-2}(U)\eta^{-2}(T) \left[1 - \frac{BC}{2} \partial_U \log \eta^2(U) \partial_T \log \eta^2(T) + O(B^2 C^2) \right], \quad (6)$$

$W_2 \simeq B+C$ and $W_3 \simeq C-B$. As expected these functions are (in case of W_1 to leading order in BC) only covariant with respect to $SO(2, 2, \mathbf{Z})$ but fail to transform correctly under shifts of the Wilson lines. Note that W_1 is the same superpotential as was postulated in ¹⁵ in the context of $(2, 2)$ orbifolds because of its correct transformation properties under $SO(2, 2, \mathbf{Z})$. We will give a more complete discussion of these issues in ⁹. Note that the BC -terms in (6) yield an alternative solution to the μ -problem ²⁰.

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